Quantum entanglement and neutron scattering experiments

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Abstract
It is shown that quantum entanglement in condensed matter can be observed with scattering experiments if the energy resolution of the experiments enables a clear separation between the elastic scattering and the scattering from the excitations in the system. These conditions are not satisfied in recent deep inelastic neutron scattering experiments from hydrogen-containing systems that have been interpreted as showing the existence of quantum entanglement for short times in, for example, water at room temperature. It is shown that the theory put forward to explain these experiments is inconsistent with the first-moment sum rule for the Van Hove scattering function and we suggest that the theory is incorrect. The experiments were performed using the unique EVS spectrometer at ISIS and suggestions are made about how the data and their interpretation should be re-examined.

1. Introduction

There has recently been much interest in the properties of systems that show quantum entanglement because many of their properties are very different from those of analogous classical systems. Some of this interest has come from the study of low-dimensional magnetic systems where systems such as linear chain antiferromagnets have non-magnetic singlet ground states that are entangled states of large numbers of particles while the excitations of these systems, spinons, are very different from the spin waves of conventional three-dimensionally ordered magnets. Other examples of entanglement are provided by cold atoms where experiments are in progress to control and exploit the entangled states so as to obtain quantum computers. Some of the most unexpected reports of nuclear quantum entanglement have come from experiments [1] performed with liquid water at room temperature and other similar experiments on hydrogen absorbed in niobium [2] and on the hydrogen in polystyrene [3]. The experiments on water studied the deep inelastic scattering of thermal neutrons from mixtures of H_2O and D_2O and reported that the scattering cross-section for the

hydrogen atoms was less than expected; the authors suggested that this was due to the quantum entanglement of the H atoms. The theory of deep inelastic neutron scattering from hydrogen has also been discussed [4] and it was suggested that the experimental results were consistent with the hydrogen atoms being entangled for short periods of time. Karlsson and Lovesey [5] and Karlsson [6] have reviewed these developments.

Most experiments that investigate quantum entanglement require very low temperatures and relatively isolated systems. It is therefore surprising that quantum effects are observed in liquid water at room temperature. In this paper these experiments are re-examined. In the next section the scattering from the simplest quantum entangled system, a spin dimer, is discussed and the conditions under which neutron scattering experiments can determine the entanglement of the two spins are obtained. Basically it is essential that the energy resolution is better than the energy splitting of the different spin configurations. This conclusion contradicts the results of the theory [4, 5] developed to explain the experiments on water and so section 3 discusses this theory and argues that the results are incorrect. In section 3 it is then argued that a corrected theory cannot explain the experimental results and is consistent with the conclusions outlined in section 2. The fourth section then reconsiders the experiments [1–3]. It is pointed out that large corrections need to be applied to the raw data to obtain $S(Q, \omega)$. It is suggested that these corrections are sufficiently uncertain, in practice, that the lack of accuracy may provide an explanation for the experimental results not involving the introduction of quantum entanglement.

2. Scattering from an entangled dimer

We consider the neutron scattering from an isolated pair of spins, $S = 1/2$, situated at the origin of space and at $R = (0, 0, r)$. The two spins interact with one another through a Heisenberg exchange interaction with $J > 0$:

$$H = J S_1 \cdot S_2.$$

The ground state is then a singlet with an energy $E = -3J/4$, and the excited states are a triplet with energies of $E = J/4$. The wavefunctions of the singlet and triplet states are

$$
\begin{aligned}
|g\rangle &= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}, \\
|1\rangle &= |\uparrow\uparrow\rangle, \\
|0\rangle &= (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2}, \\
|-1\rangle &= |\downarrow\downarrow\rangle,
\end{aligned}
\tag{1}
$$

where $|g\rangle$ is the singlet ground state wavefunction and the excited triplet states are labelled by the z-component of the total spin.

The operator for the amplitude of the neutron scattering with wavevector transfer, Q, is $A(Q) = S_1 + S_2 \exp(iQ_z r)$ and the only non-zero matrix elements of A_z connecting the states of the dimer are

$$
\begin{aligned}
\langle 0|A_z|g\rangle &= \langle g|A_z|0\rangle = (1 - \exp(iQ_z r))/2 \\
\langle 1|A_z|1\rangle &= (1 + \exp(iQ_z r))/2 \\
\langle -1|A_z|-1\rangle &= -(1 + \exp(iQ_z r))/2.
\end{aligned}
\tag{2}
$$

The cross-sections that determine the neutron scattering are proportional to the square of the matrix elements of the scattering operator and for the $\langle zz \rangle$ cross-section there are three contributions with energy transfers, E, and intensities, I, where for a system of isolated dimers

in thermal equilibrium

$$E = J, \qquad I = (1 - 3p)(1 - \cos(Q_z r))/2,$$
$$E = 0, \qquad I = p(1 + \cos(Q_z r)), \qquad (3)$$
$$E = -J, \qquad I = p(1 - \cos(Q_z r))/2.$$

p is the probability of the occupation of one of the triplet states:

$$p = \exp(-J/k_B T)/(1 + 3\exp(-J/k_B T)).$$

The rotational symmetry of the dimer in spin space shows that the $\langle xx \rangle$ and $\langle yy \rangle$ cross-sections arising from the x- and y-components of the scattering operator are the same as those from the z-component. Consequently at $T = 0$ when $p = 0$, there is no elastic scattering, while the inelastic scattering with an energy transfer of J varies in magnitude between 0 and 1 depending on the wavevector Q_z. At high temperatures, $p = 1/4$, the intensity integrated over all possible energy transfers is independent of Q_z and of magnitude $\frac{1}{2}$.

If there was no quantum entanglement, then the energy states of the dimer would be $|\uparrow\rangle$ and $|\downarrow\rangle$. The contribution of both states to S_z is $\frac{1}{2}$ and the total scattering per particle is $\frac{1}{2}$. The energy-integrated intensities of the $\langle zz \rangle$ cross-sections are identical for the two sets of states.

This analysis shows that, in principle, scattering experiments can probe whether states are entangled and suggests that a characteristic signature of entanglement is that the elastic scattering is absent at the lowest temperatures and at higher temperatures is smaller than would be obtained if the spins were not entangled. The experiments require an energy resolution that is sufficient to separate the elastic scattering from the inelastic scattering arising from the excitations of the system. If the energy resolution of the experiments is insufficient to resolve the different components of the scattering, then the energy-integrated intensity is measured and this can take very similar values whether or not the spins are entangled, as can also be obtained from sum rules. The conclusion therefore is that the effects of quantum entanglement can be observed in experiments with sufficient resolution to separate the elastic from the inelastic scattering. This implies that the resolution needs to be better than the energy of most of the excitations of the system.

These conclusions are of course well known from neutron scattering studies of highly entangled quantum systems. The ground state of an $S = 1$ antiferromagnetic chain with Heisenberg exchange interactions is a quantum singlet, the excitations are gapped and there is no elastic scattering observed at low temperatures. Superfluid ^{4}He and $S = \frac{1}{2}$ antiferromagnetic chains with Heisenberg interactions have quantum singlet ground states, while the excitations can have arbitrarily small energies. Nevertheless, neutron scattering experiments can be readily performed with energy resolutions that are better than the energies of most of the excitations and these measurements show that there is very little elastic scattering at low temperatures. Another common but less familiar example of quantum entanglement is the magnetic scattering from electrons in closed shells. Magnetic scattering from these electrons is not observed unless there is sufficient energy to excite the electrons to a higher shell. This is because they are in a quantum entangled singlet. If the electrons were not entangled their magnetic moments would each scatter the neutrons and there would be a large elastic magnetic scattering from the core electrons that would make neutron scattering studies of the magnetic properties of materials much harder.

The conclusion from the simple dimer model described above and from the experience of previous studies of quantum systems that are known to have entangled ground states is that scattering experiments do show evidence of entanglement and that the clearest signature is that the elastic scattering from the system is absent or at least small at low temperatures. More generally the experiments should be performed with an energy resolution that is better than

the energy of the excitations so that the detailed energy dependence of the scattering can be measured.

3. Deep inelastic scattering

The experiments described in the introduction were performed using deep inelastic neutron scattering in which relatively high-energy neutrons are incident on the target and individual atoms recoil essentially as free particles. Deep inelastic scattering experiments and quantum entanglement have been discussed by Karlsson and Lovesey [4, 5]. They concluded that the observed shortfall in the deep inelastic scattering intensity from the hydrogen atoms could be caused by quantum entanglement that lasts for times of about 10^{-15}–10^{-16} s. The result of their theory is however surprising because the intensity of the deep inelastic or recoil scattering is proportional to the spin-dependent incoherent neutron scattering cross-section whereas most other derivations of the cross-section for deep inelastic scattering [7] give the cross-section proportional to the total single-atom neutron scattering cross-section including both the coherent and incoherent parts. This result of Karlsson and Lovesey is unexpected because one of the systems most studied with deep inelastic scattering is liquid ^{4}He in its highly entangled superfluid state. Since the incoherent cross-section for ^{4}He is negligible, their theory predicts there is no deep inelastic scattering from liquid helium; a system in which it has frequently been observed [8]. More recently the cross-section has been discussed [5] in which the scattering has been taken to be the total scattering but reduced by a factor of about $\frac{1}{4}$ compared with the cross-section for the total scattering. There is no evidence of this reduction in the cross-section from liquid ^{4}He.

Two further arguments suggest that the conclusions of [4, 5] are incorrect. The first, and probably the most convincing, is that the theory is inconsistent with the first-moment sum rule [6] that states that the first energy moment of the Van Hove scattering function $S(Q, \omega)$ for a monatomic system is $h^2 Q^2/(2M)$ where M is the mass of the scattering nucleus. This is derived by taking the commutator of the density operator with the Hamiltonian of the system and depends only on the potential energy terms in the Hamiltonian being independent of the momenta of the particles. It is independent of the exact state of the system whether it is a solid, liquid or gas and also whether or not there is quantum entanglement. For the deep inelastic scattering at large Q, the sum rule implies that the energy-integrated scattering is proportional to the total atom cross-section if the scattering is centred at the recoil energy of $h^2 Q^2/(2M)$. If the intensity at the recoil energy is lower than the total atom cross-section, the sum rule implies that there must be some additional scattering at both lower and higher energies. There is no such scattering predicted by the theory. Furthermore, since the recoil energy is much higher than the energy of the excitations, there are no processes that can give rise to scattering at higher energies. We conclude that the sum rule when applied to deep inelastic scattering shows that the total atom scattering cross-section determines the intensity of the recoil peak independently of the state of the system.

A detailed study of [4] and [5] and a comparison with the development of the theory of the scattering by homonuclear diatomic molecules [7] has also been made. The crucial difference of the two developments occurs at the end of section 3 of [4] and in [4] results in the scattering being proportional to only the incoherent cross-section. It is argued that if two nuclei are initially in a total nuclear spin state, J, then after the deep inelastic scattering they will be in the state J' where $J' = J + 1$ or $J - 1$ but not J. Since coherent scattering is nuclear spin independent this conclusion inevitably leads to the scattering depending only on the incoherent scattering cross-section. The exclusion of the $J' = J$ terms is obtained as a consequence of the orthogonality of the initial and final wavefunctions of the system. The relevant matrix elements

of the scattering differ however from those of the orthogonality condition by the presence of $\exp(i\boldsymbol{Q} \cdot \boldsymbol{R})$ where $\boldsymbol{Q}$ is the wavevector transfer, $\boldsymbol{k}$ in [4]. Since $\boldsymbol{Q}$ is very similar to $\boldsymbol{p}'$, the wavevector of the recoiling particle, the states can be orthogonal but yet the matrix elements of the scattering operator can be non-zero. Consequently the terms with $J' = J$ should be included as they are in the analysis of the scattering from diatomic molecules in [6]. If these terms are included in equation (4.5) of [4], there are two additional terms. The first term is proportional to the incoherent scattering cross-section:

$$(\sigma_{inc}/4\pi)|T_1 + \zeta T_2|^2(J(J+1))/(4I(I+1)), \tag{4}$$

while the second term is proportional to the coherent scattering cross-section:

$$(\sigma_{coh}/4\pi)|T_1 + \zeta T_2|^2. \tag{5}$$

The notation follows [4]; ζ is the phase factor $(-1)^J$, I is the nuclear spin and J is the spin of the two-atom-entangled wavefunction. These results are different from those given before [4, 5] and correspond to the difference of the J-terms being included as well as the factor of 2 in the notation.

The overlap integrals, T_1 and T_2, require further discussion and are defined in equation (3.9) of [4] as

$$T_2 = \int d\boldsymbol{R}\, \psi^*(\boldsymbol{R})\varphi_2(\boldsymbol{R}), \tag{6}$$

where $\varphi_2(\boldsymbol{R})$ is the wavefunction of one of the particles in the initial state and $\psi(\boldsymbol{R})$ is the wavefunction after the scattering of the particle that does not recoil with a large kinetic energy. The basic principle of deep inelastic scattering is that only one particle is given a large kinetic energy while the others are largely undisturbed. There may later be a readjustment of the system but this does affect the initial recoil. Initially it is assumed that one particle has wavefunction φ_1 and the other φ_2 and their overlap is small [3]; the wavefunction ψ must be either φ_1 or φ_2. One of the overlap integrals is then 1 while the other is zero, so

$$|T_1 + \zeta T_2|^2 = 1. \tag{7}$$

The scattering can then be calculated by adding equation (4) to (4.5) of [4] and using equation (7) to give the spin-dependent scattering as proportional to $\sigma_{inc}/4\pi$ and then adding the spin-independent scattering, equation (5), $\sigma_{coh}/4\pi$ to give the total scattering proportional to the total atom scattering cross-section as expected from the sum rule analysis given above.

4. The experiments

Three experiments have been reported that show a smaller deep inelastic scattering than expected from conventional theory. The first of these [1] was on mixtures of H_2O and D_2O and reported that the relative intensity of the deep inelastic scattering from H and D atoms did not vary as expected with the concentration of H_2O and D_2O. The second experiment [2] studied Nb hydrides and found that the ratio of the scattering from the H atoms to that from the Nb atoms was smaller than expected. The third experiment used a sample of polystyrene and found that the ratio of the scattering from the H atoms to that from the C atoms was smaller than expected.

All of these experiments were performed using the EVS spectrometer at the ISIS pulsed neutron facility. This is a unique instrument and consequently none of the experiments have been repeated on other instruments. The instrument is an inverse time-of-flight spectrometer designed so as to make use of the epithermal neutron flux from the ISIS pulsed source.

The scattered neutrons are detected by the filter difference technique in which the scattered neutron flux is measured with no filter and then with filters that absorb neutrons of a particular energy. Differencing the fluxes then enables the intensity of neutrons scattered with that particular energy to be obtained. The experiments used either Au filters that absorb neutrons with an energy of 4.908 eV within an energy width of 0.138 eV or U filters that absorb neutrons with an energy of 6.771 eV within a width of 0.062 eV. The time of flight of the neutrons from the accelerator target to the detector is measured so that the incident energy of the neutrons and hence their energy transfer can be calculated from the known energy of the scattered neutrons and the distances from the neutron source to the sample and from the sample to the detectors. The detector bank contained 16 detectors at different angles of scattering and the angles between the incident and scattered beams were typically varied between 30° and 80°.

The results of the experiment are reported as time-of-flight spectra for particular scattering angles and show two or three peaks corresponding to the scattering from H atoms, from D atoms and from heavy atoms. The peaks are always positioned accurately at the times expected for the free-atom recoil of the respective atoms. The widths of the peaks depend on the kinetic energies of the particles before the neutron scattering and on the experimental resolution, while the intensities of the peaks depend on the scattering cross-sections. There are however large corrections that need to be applied to the data before the results can be compared with the theoretical predictions.

One correction arises from the definition of the cross-section. Theory calculates the scattering for unit incident flux into a solid angle $d\Omega$ and an energy interval dE and for a monatomic system the cross-section is proportional to $(k_1/k_0)S(Q, \omega)$ where k_0 and k_1 are the wavevectors of the incident and scattered neutrons while $S(Q, \omega)$ is the Van Hove scattering function. In the inverse time-of-flight geometry the detector system has a constant angular acceptance and energy resolution because the detector is fixed in angle and energy and the resolution in energy is determined by the resonance width of the absorbing foil. The incident neutron flux varies with the incident neutron energy, E_0, and has a distribution $N(E_0) \, dE_0$ where for the epithermal neutrons from a pulsed source $N(E)$ is approximately c/E where c is a constant. The observed energy distribution is then proportional to $N(E_0)(k_1/k_0)S(Q, \omega)$. The spectra are however recorded and plotted in the papers as a function of time of flight instead of energy and so the incident flux must be converted to a time distribution $N(t) = N(E) \, dE/dt$ [9]. The observed intensity is then proportional to $N(E_0)k_1k_0^2 S(Q, \omega)$.

For any particular detector, while the time of flight varies both the energy transfer E and the wavevector transfer Q vary. As a result, the integrated intensity of a peak in the spectra is not the same as that obtained in a constant-Q scan that is easily compared with theory. The correction was calculated for a conventional time-of-flight instrument by Waller and Fröman [10] but has not, to our knowledge, been explicitly obtained for an inverse time-of-flight instrument. The correction shows that the observed intensity is multiplied by a factor of $1/J$ compared with the intensity that would be observed in a constant-Q experiment, where J is the Jacobian of the transformation from the time-of-flight scan to the constant-Q scan. For deep inelastic scattering from atoms of mass M the Jacobian for a conventional time-of-flight experiment is

$$J = 1 + (m/M)(1 - (k_0/k_1)\cos(\varphi)) \qquad (8)$$

while for an inverse time-of-flight instrument the Jacobian is

$$J = 1 - (m/M)(1 - (k_1/k_0)\cos(\varphi)) \qquad (9)$$

where m is the mass of the neutron and φ is the scattering angle. For the case of H atoms, $m = M$ and J is particularly simple and significant being $2 - (k_0/k_1)\cos(\varphi)$ and $(k_1/k_0)\cos(\varphi)$ respectively. The relative scattering cross-sections can then be obtained from the intensities of observed peaks if the observed intensities are multiplied by $J/(N(E_0)k_0^2)$ before comparing
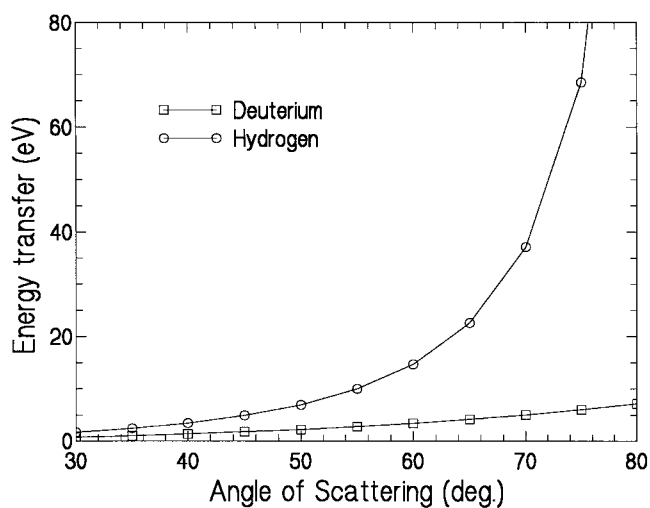
Figure 1. Neutron energy transfer for deep inelastic scattering from hydrogen and deuterium measured using the EVS spectrometer with Au absorbers as a function of the angle of scattering.
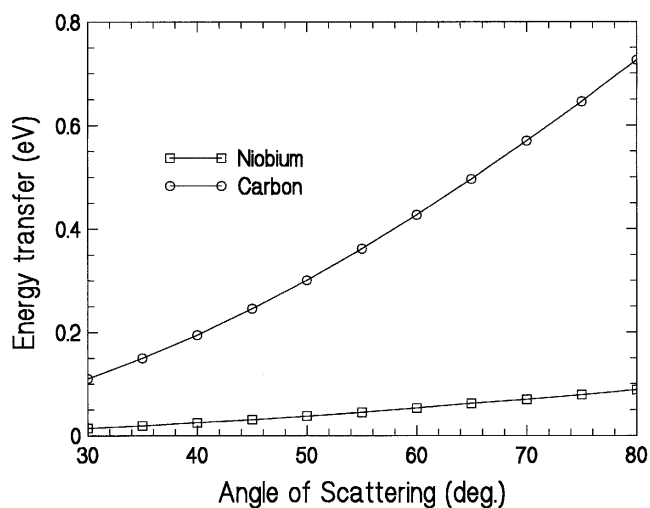

Figure 2. Neutron energy transfer for deep inelastic scattering from carbon and niobium measured using the EVS spectrometer with Au absorbers as a function of the angle of scattering.

with the theoretical results. If $N(E)$ is directly proportional to $1/E$, then this correction factor is proportional to J.

The size of these correction factors has been estimated for the experiments described above and the results are shown in figures 1–3. The energy transfers are shown in figure 1 for deep inelastic scattering from hydrogen and deuterium and from carbon and niobium in figure 2. For both cases the filters were assumed to be gold foils with a resonance energy of 4.908 eV. For the case of hydrogen the incident neutron energy varies between 6.5 eV at 30° and 163 eV at 80°, whereas for deuterium the corresponding energies are 5.6 and 12 eV and for carbon they are 5.0 and 5.6 eV respectively. The analysis of the data for the hydrogen scattering is then dependent on correctly taking account of the change in the incident neutron flux when
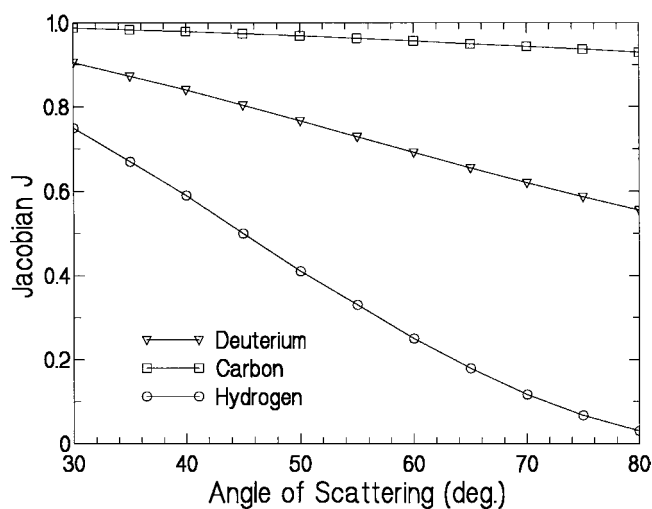
Figure 3. The Jacobian for deep inelastic neutron scattering from carbon, deuterium and hydrogen measured using the EVS spectrometer with Au absorbers as a function of the angle of scattering.

the incident energy varies by possibly a factor of 20 or more. Figure 3 shows the variation of the Jacobian factor, J, as the scattering angle varies for scattering by hydrogen, deuterium and carbon. In the case of carbon, J varies by only 6% between 30° and 80° and even less for scattering from niobium. The variation is much larger for scattering by deuterium and even larger, a factor of 25, for hydrogen. The final correction then depends on these two corrections for the variation of the incident neutron flux and the Jacobian. Both are large corrections and they largely cancel one another out. Nevertheless, since their physical origins are different, there is no reason to expect their respective errors to be cancelling.

The analysis of the experimental data assumed that the profile of the deep inelastic scattering was a Gaussian and convoluted this with a Lorentzian experimental resolution function. The energy of the peaks was held fixed at the recoil energy and the widths and intensities of the deep inelastic scattering varied to fit the experimental data. This procedure should include the corrections for both the varying incident neutron flux and the Jacobian. It is, however, unclear how accurate the calculations are. For example, there is little information readily available about how the energy dependence of the incident neutron flux was determined and what measurements have been made to check the accuracy of these measurements. The results reported for hydrogen in niobium [2] and for hydrogen in polystyrene [3] for scattering angles above 50° show a decrease in the intensity of the hydrogen scattering with increasing angle. This could result if the incident neutron flux is smaller at high energies than the flux included in the calculations. At smaller scattering angles the results from these two experiments are different, with the scattering from the niobium hydride increasing to the expected value as the angle of scattering decreases to about 40° while for polystyrene the cross-section remains constant at about 14% less than the expected scattering. At these angles the energy transfer to the carbon atoms is less than 0.25 eV and to the hydrogen atoms is less than 4.9 eV and the scattering peaks from the hydrogen and the carbon overlap [9]. These energy transfers are not much larger than the energies of the stretching vibrations of the C–H bonds, 0.4 eV, and so the adequacy of the Gaussian approximation for the scattering theory and the difficulty of separating the scattering from the different atoms could explain why these results do not agree with one another or with the conventional theory.

The experiments on niobium containing both hydrogen and deuterium gave results for the hydrogen scattering cross-section at scattering angles above 50° that are similar to those for the sample containing only hydrogen. At smaller angles it is difficult to distinguish between the hydrogen, deuterium and niobium scattering as discussed above. The ratio of the deuterium scattering to the niobium scattering is at all angles about 10% less than expected [2]. This discrepancy may arise from a failure of the deep inelastic scattering approximation to describe the niobium scattering because the energy transfers are not much larger than possible phonon energies.

The scattering from mixtures of H_2O and D_2O was analysed differently [1]. The results obtained with the Au absorber and scattering angles between 50° and 75° were used to obtain the ratio of the H and D cross-sections averaged over the different scattering angles. The data between 30° and 60° obtained with the U absorber were analysed similarly. The results for the ratio of the hydrogen cross-section to the deuterium cross-section decreased from about 10.7 (the expected value) for close to pure D_2O to about 6.7 for a mixture with 30% of D_2O. Figure 1 of [1] shows the observed time-of-flight spectra for scattering angles of 69° and 80% of D_2O; the ratio of the scattering in the hydrogen and deuterium peaks is about 1.8–1. This ratio increases to 7.2–1 for a concentration of 50% and 17–1 for a concentration of 30%. At 69° the scattering from the deuterium overlaps both with the scattering from the oxygen at long times and the scattering from the hydrogen at short times, figure 1. There will be more overlap of the peaks at smaller scattering angles. Furthermore, since the O–H stretching mode has an energy of about 0.4 eV there could be deviations from the Gaussian line-shape for the deep inelastic scattering. It is therefore very difficult to be confident of the background under the deep inelastic peaks and of the accuracy with which the different components can be separated and hence the accuracy of the ratio of the scattering cross-sections of H and D, particularly for samples with less than 50% of deuterium.

5. Conclusions

The objective of this paper has been to examine the conditions under which it is possible to study quantum entanglement with neutron scattering techniques and to compare the conclusions with the recent results obtained using the EVS spectrometer at ISIS on hydrogen-containing materials. By consideration of an $S = \frac{1}{2}$ spin dimer it is suggested that quantum entanglement is only observable by scattering techniques if the energy resolution of the scattering experiment is less than the energy difference of the quantum entangled ground state and most of the excitations of the system. In some systems it is also an advantage if the temperature is sufficiently low that there are only a few of the excitations excited. These conclusions are consistent with experiments on quantum magnets, superfluid helium and the absence of magnetic scattering from atoms with electronic closed shells.

The deep inelastic scattering experiments that have reported evidence of quantum entanglement do not satisfy these conditions. The energy resolution in the experiments is typically 0.14 eV or more. The energy of the excitations corresponding to the spin orientation of the H nuclei is much less. For the hydrogen molecule the energy difference of the ortho- and para-states is 0.024 eV and results mainly from the different rotational energies of the two states. For the hydrogen nuclei in water the symmetry is less and the difference in the energy of states with different nuclear spins is probably much less than for hydrogen molecules. Consequently the theory and the experiments have been examined in detail.

The theory [4, 5] that in general terms supports the reported experimental results has been discussed and is flawed. The most convincing argument for this is that it is inconsistent with the first-moment sum rule for neutron scattering. Nevertheless, it is shown in section 3 that

the failure of the theory results from an incorrect assumption about the orthogonality of the initial and final states of the system and that when this is corrected the scattering cross-section is consistent with that of the conventional theory.

It is more difficult to identify whether the experiments are flawed. EVS is a unique spectrometer of unconventional design that enables neutron scattering experiments to be performed using incident neutrons of energies very different from those of almost all other neutron scattering experiments from condensed matter. It is a very important instrument in opening up new opportunities to do these experiments and the experiments discussed here are the first ones that have made quantitative use of the intensities of the deep inelastic scattering. The uniqueness of the instrument and the impossibility of repeating the experiments elsewhere make it necessary to examine the experiments very carefully. In section 4 the experiments were described and points of possible contention raised. There are large corrections to the raw data needed before the cross-sections are obtained. These corrections are understood but there is no independent check that they are all applied sufficiently accurately. More examination is required of the normalization of the data to a constant incident neutron flux. This requires the determination of the flux of incident neutrons for energies between 1 and 150 eV. The second point is the difficulty of separating the scattering from different atomic masses. This is particularly the case for experiments at the lower angles of scattering and in the experiments where both hydrogen and deuterium atoms are present. The necessary accuracy in the intensities of the peaks requires a detailed knowledge of the wings of the deep inelastic scattering profiles for both the H atoms and for the other atoms in the samples and the nature of the background under the peaks. The third possible difficulty is that at the smaller angles and for the heavier elements the energy transfers are comparable with the energies of the hydrogen stretching vibrations. The accuracy of the Gaussian approximation for the deep inelastic scattering is then uncertain.

In conclusion, it is unlikely that evidence of quantum entanglement has been observed by neutron deep inelastic scattering techniques in systems containing hydrogen at room temperature. It is possible that the observed effects arise from problems in the analysis of the data from the unique instrument used for the measurements. We have suggested a number of possible reasons for the problems and hope that this paper will lead to further experiments and a clarification of whether these important results are correct.

Acknowledgments

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